Factored MDPs for Detecting Topics of User Sessions

Maryam Tavakol & Ulf Brefeld

Knowledge Mining & Assessment
brefeld@cs.tu-darmstadt.de
Traditional Item-to-Item

- A user views the following item:

- Task: Recommend an item that is likely to be clicked
- But: What’s the reason for viewing that item?
Session-to-Item

- A user views the following sequence of items:

- What is the user’s goal of the session?
- Take a content-based approach!
Attribute View

- Category: Shirt, Shirt, Shirt, Shirt
- Colour: Dark Blue, Black, Dark Brown, Black
- Gender: Women, Women, Unisex, Women
- Price: Cheap, Expensive, Expensive, Cheap

$t = 1, t = 2, t = 3, t = 4$
Attribute View

Category: Shirt, Shirt, Shirt, Shirt

Colour: Dark Blue, Black, Dark Brown, Black

Gender: Women, Women, Unisex, Women

Price: Cheap, Expensive, Expensive, Cheap

$\mathbf{t} = 1, \mathbf{t} = 2, \mathbf{t} = 3, \mathbf{t} = 4$

Topic: Shirt, Dark Colours, Women, Any
Markov Decision Processes

- 4-tuple $\langle S, A, R, P \rangle$
- Set of states $S$ (last $k$ viewed items/user clicks)
- Set of actions $A$ (items)
- Reward function $R : S \times A \rightarrow \mathbb{R}$ (positive for clicks on recommended items)
- Transition probabilities $P : S \times A \times S \rightarrow [0, 1]$
Factored MDPs

- States decompose into **state variables**

\[ S = X_1 \times X_2 \times \ldots \times X_n \]

- Factorisation of probability distribution

\[
P(S'|S, a) = \prod_{j=1}^{n} P(X'_j|parent(X'_j), a)\]

\text{value of attribute } j \text{ given all attributes of previous items}
Attribute Independence

complete model infeasible

(category, brand, price)

\[ \mathcal{O}(d^n) \]
Attribute Independence

complete model infeasible

\[ \mathcal{O}(d^n) \]

exploit independence

\[ \mathcal{O}(dn) \]

(see theorem in the paper)
Exact and Approximate fMDPs

- **Exact** \( P(x'|s, x) \) estimated by Maximum Likelihood

- **Approximate** (Shani et al., JMLR 2005)
  approximate \( P(x|s, x) \approx \alpha P(x|s) \) and \( P(x|s, x') \approx \beta P(x|s) \)

- **Optimisation by value iteration**

\[
Q(s_t, x_t) = R(s_t, x_t) + \gamma \sum_{s_t'} P(s_t'|s_t, x_t) V^*(s_t')
\]

value of recommending attribute realisation \( x_t \) when in state \( s_t \)
Topic Detection

- Use min-max normalisation of $Q(s_j, x_j)$ values

$$q(x_j = x_j | s_j) = \frac{Q(s_j, x_j) - \min_{x'_j}[Q(s_j, x'_j)]}{\max_{x'_j}[Q(s_j, x'_j)] - \min_{x'_j}[Q(s_j, x'_j)]}$$

- Thresholding important!

<table>
<thead>
<tr>
<th>colour</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(black</td>
<td>s) = 0.8</td>
</tr>
<tr>
<td>q(blue</td>
<td>s) = 0.7</td>
</tr>
<tr>
<td>q(green</td>
<td>s) = 0.4</td>
</tr>
<tr>
<td>q(red</td>
<td>s) = 0.2</td>
</tr>
</tbody>
</table>
Topic Detection

- **Use min-max normalisation of** $Q(s_j, x_j)$ **values**

\[
q(x_j = x_j | s_j) = \frac{Q(s_j, x_j) - \min_{x_j'} [Q(s_j, x_j')]}{\max_{x_j'} [Q(s_j, x_j')] - \min_{x_j'} [Q(s_j, x_j')]} \]

- **Thresholding important!**

<table>
<thead>
<tr>
<th>colour</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(black</td>
<td>s) = 0.8</td>
</tr>
<tr>
<td>q(blue</td>
<td>s) = 0.7</td>
</tr>
<tr>
<td>(red</td>
<td>s) = 0.4</td>
</tr>
<tr>
<td>(green</td>
<td>s) = 0.2</td>
</tr>
</tbody>
</table>

**topic = {black, blue, expensive, cheap, sale}**
Empirical Evaluation

- Transaction data from Zalando
- About 1.7 million user sessions
- > 24 million clicks
- Attributes: category, colour, gender, price
- User parameters optimised by model selection
Impact of Threshold

- Size of topics decreases wrt threshold
- Topic accuracy decreases wrt threshold
**Accuracies (Small-Scale)**

<table>
<thead>
<tr>
<th>k</th>
<th>joint</th>
<th>colour</th>
<th>gender</th>
<th>category</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>33.69</td>
<td>49.78</td>
<td>92.24</td>
<td>78.52</td>
<td>63.96</td>
</tr>
<tr>
<td>3</td>
<td>37.70</td>
<td>52.98</td>
<td>92.31</td>
<td>79.50</td>
<td>65.06</td>
</tr>
<tr>
<td>2</td>
<td>37.65</td>
<td>52.15</td>
<td>92.22</td>
<td>79.68</td>
<td>64.24</td>
</tr>
<tr>
<td>1</td>
<td>28.06</td>
<td>44.31</td>
<td>91.85</td>
<td>79.01</td>
<td>56.28</td>
</tr>
</tbody>
</table>

**Markov Process**

<table>
<thead>
<tr>
<th>k</th>
<th>joint</th>
<th>colour</th>
<th>gender</th>
<th>category</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>67.53</td>
<td>85.61</td>
<td>95.00</td>
<td>90.70</td>
<td>78.68</td>
</tr>
<tr>
<td>3</td>
<td>69.56</td>
<td>93.94</td>
<td>95.21</td>
<td>93.36</td>
<td>72.01</td>
</tr>
<tr>
<td>2</td>
<td>40.62</td>
<td>45.96</td>
<td>95.30</td>
<td>94.90</td>
<td>78.39</td>
</tr>
<tr>
<td>1</td>
<td>16.47</td>
<td>28.37</td>
<td>95.31</td>
<td>95.28</td>
<td>46.55</td>
</tr>
</tbody>
</table>

**MDP (exact)**

<table>
<thead>
<tr>
<th>k</th>
<th>joint</th>
<th>colour</th>
<th>gender</th>
<th>category</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>75.33</td>
<td>81.92</td>
<td>94.65</td>
<td>90.05</td>
<td>92.38</td>
</tr>
<tr>
<td>3</td>
<td>89.52</td>
<td>92.95</td>
<td>94.83</td>
<td>92.81</td>
<td>94.48</td>
</tr>
<tr>
<td>2</td>
<td>93.69</td>
<td>95.12</td>
<td>94.97</td>
<td>94.45</td>
<td>95.00</td>
</tr>
<tr>
<td>1</td>
<td>94.14</td>
<td>95.25</td>
<td>94.98</td>
<td>94.82</td>
<td>94.97</td>
</tr>
</tbody>
</table>

**MDP (approx)**

<table>
<thead>
<tr>
<th></th>
<th>joint</th>
<th>colour</th>
<th>gender</th>
<th>category</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDA</td>
<td>1.65</td>
<td>11.76</td>
<td>85.89</td>
<td>52.8</td>
<td>21.14</td>
</tr>
</tbody>
</table>

- longer chains better but data too sparse
- estimation of $\alpha$ and $\beta$ better for shorter chains
Accuracies (Large-Scale)

<table>
<thead>
<tr>
<th>k</th>
<th>joint</th>
<th>colour</th>
<th>gender</th>
<th>category</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>39,56</td>
<td>53,50</td>
<td>89,70</td>
<td>77,93</td>
<td>71,25</td>
</tr>
<tr>
<td>3</td>
<td>39,53</td>
<td>52,83</td>
<td>89,70</td>
<td>78,09</td>
<td>71,04</td>
</tr>
<tr>
<td>2</td>
<td>38,37</td>
<td>50,78</td>
<td>89,57</td>
<td>77,94</td>
<td>71,09</td>
</tr>
<tr>
<td>1</td>
<td>30,82</td>
<td>42,37</td>
<td>89,15</td>
<td>77,29</td>
<td>70,02</td>
</tr>
<tr>
<td>4</td>
<td>88,3</td>
<td>91,09</td>
<td>92,61</td>
<td>90,88</td>
<td>92,19</td>
</tr>
<tr>
<td>3</td>
<td>91,13</td>
<td>92,73</td>
<td>92,45</td>
<td>92,04</td>
<td>92,56</td>
</tr>
<tr>
<td>2</td>
<td>91,48</td>
<td>92,82</td>
<td>92,46</td>
<td>92,37</td>
<td>92,49</td>
</tr>
<tr>
<td>1</td>
<td>91,53</td>
<td>92,85</td>
<td>92,4</td>
<td>92,39</td>
<td>92,55</td>
</tr>
</tbody>
</table>

Markov Process

MDP (approx)

LDA

more data diminish effect of shorter chains
Topic-driven Recommendations

- Turn $Q$-values into probabilities (softmax)

$$\Pr(\mathcal{X}_j = x_j | s_j) = \frac{\exp\{Q(s_j, x_j)\}}{\sum_{x'_i} \exp\{Q(s_j, x'_i, j)\}}$$

- Rank items $i$ according to sum of log-probabilities (exploiting independence)

$$\text{score}(i; s) = \prod_{j=1}^{n} P(\mathcal{X}_j = x_j | s_j) \propto \sum_{j=1}^{n} \log P(\mathcal{X}_j = x_j | s_j)$$
Topic-driven Recommendations

- Topic-driven recommendation outperforms traditional CF/MF approaches
Topic-driven Recommendations

Topic-driven recommendation outperforms traditional CF/MF approaches
Topic-driven Recommendations

- Topic-driven recommendation outperforms traditional CF/MF approaches.

- Collaborative filtering w matrix factorisation

- Collaborative filtering w topic models

- Random product

- Most popular

- Copy attributes of previous item

- Approx MDP w different Markov assumptions
Topic-driven Recommendations

- Topic-driven recommendation outperforms traditional CF/MF approaches
Conclusion

- Topic detection for user sessions
  - Sessions-based approach = short-term interests
  - Exploit sequential nature of the data (MDP)
  - Content-based (factorise over attributes)
- Empirically outperform traditional CF/MF recommenders and straw men

Maryam Tavakol: tavakol@cs.tu-darmstadt.de
Variance of Topics

- Uncertainty decreases in length of session
- Markov assumption influences convergence